Gradient, Divergence, Curl, and Laplacian Operations

Cartesian Coordinates

$$\nabla V = \hat{a}_x \frac{\partial V}{\partial x} + \hat{a}_y \frac{\partial V}{\partial y} + \hat{a}_z \frac{\partial V}{\partial z}$$

$$\partial$$

$$\nabla \cdot \overline{A} = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}$$

$$\nabla \times \overline{A} = \begin{vmatrix} \hat{a}_x & \hat{a}_y & \hat{a}_z \\ \dfrac{\partial}{\partial x} & \dfrac{\partial}{\partial y} & \dfrac{\partial}{\partial y} \\ A_x & A_y & A_z \end{vmatrix}$$

$$\nabla^2 V = \frac{\partial^2 V}{\partial^2 x} + \frac{\partial^2 V}{\partial^2 y} + \frac{\partial^2 V}{\partial^2 z}$$

Cylindrical Coordinates

$$\nabla V = \hat{a}_\rho \frac{\partial V}{\partial \rho} + \hat{a}_\phi \frac{\partial V}{\rho \partial \phi} + \hat{a}_z \frac{\partial V}{\partial z}$$

$$\nabla \cdot \overline{A} = \frac{1}{\rho} \frac{\partial \left(\rho A_\rho \right)}{\partial \rho} + \frac{\partial A_\phi}{\rho \partial \phi} + \frac{\partial A_z}{\partial z}$$

$$\nabla \times \overline{A} = \hat{a}_\rho \left(\frac{\partial A_z}{\rho \partial \phi} - \frac{\partial A_\phi}{\partial z} \right) + \hat{a}_\phi \left(\frac{\partial A_\rho}{\partial z} - \frac{\partial A_z}{\partial \rho} \right) + \hat{a}_z \frac{1}{\rho} \left(\frac{\partial \left(\rho A_\phi \right)}{\partial \rho} - \frac{\partial A_\rho}{\partial \phi} \right)$$

$$\nabla^2 V = \frac{1}{\rho} \frac{\partial}{\partial \rho} \left(\rho \frac{\partial V}{\partial \rho} \right) + \frac{1}{\rho^2} \frac{\partial^2 V}{\partial \phi^2} + \frac{\partial^2 V}{\partial z^2}$$

Spherical Coordinates

$$\nabla V = \hat{a}_r \frac{\partial V}{\partial r} + \hat{a}_\theta \frac{\partial V}{r \partial \theta} + \hat{a}_\phi \frac{1}{r \sin\theta} \frac{\partial V}{\partial \phi}$$

$$\nabla \cdot \overline{A} = \frac{1}{r^2} \frac{\partial}{\partial r} \left(r^2 A_r \right) + \frac{1}{r \sin\theta} \frac{\partial \left(A_\theta \sin\theta \right)}{\partial \theta} + \frac{1}{r \sin\theta} \frac{\partial A_\phi}{\partial \phi}$$

$$\nabla \times \overline{A} = \hat{a}_r \frac{1}{r \sin\theta} \left(\frac{\partial \left(A_\phi \sin\theta \right)}{\partial \theta} - \frac{\partial A_\theta}{\partial \phi} \right) + \hat{a}_\theta \frac{1}{r} \left(\frac{1}{\sin\theta} \frac{\partial A_r}{\partial \phi} - \frac{\partial \left(r A_\phi \right)}{\partial r} \right) + \hat{a}_\phi \frac{1}{r} \left(\frac{\partial \left(r A_\theta \right)}{\partial r} - \frac{\partial A_r}{\partial \theta} \right)$$

$$\nabla^2 V = \frac{1}{r^2} \frac{\partial}{\partial r} \left(r^2 \frac{\partial V}{\partial r} \right) + \frac{1}{r^2 \sin\theta} \frac{\partial}{\partial \theta} \left(\sin\theta \frac{\partial V}{\partial \theta} \right) + \frac{1}{r^2 \sin^2\theta} \frac{\partial^2 V}{\partial \phi^2}$$